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Structure of a Spinning Point Particle at Rest

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The momentum, angular momentum, and energy of a spinning particle each consists of two terms, as if the particle were composed of two other particles that move coherently in different orbits about their common center of mass. The conditions that the momentum terms cancel each other and that the orbital angular momenta should be quantized lead to a series of quantized mass-energy levels and mass ratios that describe the relevant experimental data to better than 0.25%. The correlated particles move as if they have a potential energy that is constant for very small distances from the fixed center of mass and that increases linearly for large distances, but no quarks or gluons are postulated. Baryons and mesons are linked to each other, as are baryons and antibaryons.

1. MOMENTUM, ENERGY, AND MASS

It is usually assumed that the momentum of an object is its mass m times the velocity $\mathbf{v}$ of its center of mass, with m replaced by the relativistically increased mass $m\gamma$ when it is necessary to take that effect into account. It is also assumed that the total energy is $m\gamma c^2$. If the radius of the object is small compared with other distances that occur in the problem, as for the earth in its orbit around the sun, it is a good approximation to treat it as a "particle"—a point object—coincident with its center of mass. However, if the particle has a spin $\boldsymbol{\sigma}$, its momentum is not simply $m\gamma\mathbf{v}$ and its energy is not simply $m\gamma c^2$—other terms, proportional to the spin, must be added to these expressions. These extra terms are completely negligible for the earth ($\sim 10^{-15}$), and even for an electron in an atom they are small [$\sim(Z\alpha)^2$], although large enough to account for the fine structure of spectral lines. However, the smaller the region in which the particle moves, the more important these terms become. For a particle confined to a region

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of dimensions $\sim\sigma/mc$, these extra terms, often neglected, are comparable in magnitude to the usual ones.

It has been known for decades that energy and electric charge transform differently under a Lorentz transformation and that if a spinning particle is charged and is moving with velocity $\mathbf{v}$ relative to an observer, its center of mass lies outside of it, being separated from it by the vector $(\mathbf{v}\times\boldsymbol{\sigma})/E$, where E is its energy (Moller, 1952, pp. 170–173). If this vector remains constant in magnitude and direction for a free particle, this means that the center of charge (sometimes called the proper center of mass) and the center of mass, coincident when the particle is at rest, move parallel to each other, separated by this distance.

However, there is no reason to assume that this vector must remain fixed in direction. The center of mass of a free particle can move with constant (including zero) velocity, and the particle, with its charge, if any, can move in a circle around it with velocity $\mathbf{v}$. If the center of mass is at rest, there is no orbital angular momentum, so that the spin σ must be constant. With the particle moving in a circle around its center of mass and normal to σ, the radius of the circle is then $v\sigma(Mc^2)^{-1}$ and the angular velocity ω is given by $|\sigma\omega| = Mc^2$.

In more detail, the extra terms added to the momentum $\mathbf{P}$ and energy E, and therefore to the rest mass M, are given by[3]

$$\begin{aligned}\mathbf{P} &= m\gamma\mathbf{v} + \mathbf{P}_\sigma \\ E &= m\gamma c^2 + \mathbf{v}\cdot\mathbf{P}_\sigma \\ &= \mathbf{v}\cdot\mathbf{P} + mc^2\gamma^{-1}\end{aligned} \tag{1}$$

so that

$$\begin{aligned}M^2c^4 &= E^2 - P^2c^2 \\ &= m^2c^4 - \mathbf{P}_\sigma^{\,2}c^2 + (\mathbf{v}\cdot\mathbf{P}_\sigma)^2\end{aligned} \tag{2}$$

where

$$\mathbf{P}_\sigma = -\gamma^2(\boldsymbol{\sigma}\times d\mathbf{v}/dt)c^{-2} \tag{3}$$

For the special case $\mathbf{P}=0$, it follows that $\boldsymbol{\sigma}\cdot\mathbf{v}=0$, $\gamma^2=\text{const}$. Thus the particle moves with constant speed in a plane normal to the constant spin $\boldsymbol{\sigma}$ and with constant-magnitude acceleration normal to $\mathbf{v}$. If $\mathbf{v}=\boldsymbol{\omega}\times\mathbf{r}$, it follows that

$$-\boldsymbol{\sigma}\cdot\boldsymbol{\omega} = mc^2\gamma^{-1} = Mc^2 \tag{4}$$

[3]These equations are given implicitly in Bhabha and Corben (1941), Aldinger *et al.* (1983), Hughes and Wu (1977), and Corben (1984), and explicitly in Corben (1961*a*, equations (24), 1961*b*, 1993).

This is the classical limit of *Zitterbewegung*. Thus $\boldsymbol{\omega}$ is antiparallel to $\boldsymbol{\sigma}$ and

$$\mathbf{r} = (\boldsymbol{\sigma} \times \mathbf{v})(Mc^2)^{-1} \tag{5}$$

as before ($\mathbf{r}$ is the vector from the fixed center of mass to the particle).

2. FICTITIOUS MASS

Because $\mathbf{P}$ is the sum of two momenta and E is the sum of two energies, we may write equations (1) to *look* like that, defining μ, Γ, $\mathbf{V}$ by

$$\mathbf{P}_\sigma = \mu\Gamma\mathbf{V}, \qquad \mathbf{v}\cdot\mathbf{P}_\sigma = \mu\Gamma c^2, \qquad \Gamma^{-2} + V^2/c^2 = 1 \tag{6}$$

so that

$$\mathbf{V}\cdot\mathbf{v} = c^2 \tag{7}$$

For $\mathbf{P} = 0$ we also have

$$\mu\Gamma\mathbf{V} = -m\gamma\mathbf{v} \tag{8}$$

so that

$$\mu\Gamma = -m\gamma\beta^2 \tag{9}$$

as expected, and

$$\mathbf{v} = \beta^2\mathbf{V}, \qquad \mathbf{V} = B^2\mathbf{v} \tag{19}$$

where $V^2/c^2 = B^2$. If $\mathbf{v} < c$, then $\mathbf{V} > c$. Since $\mathbf{V}$ and $\mathbf{v}$ are parallel, we have

$$\mathbf{V} = \boldsymbol{\omega} \times \mathbf{R} = \beta^{-2}\mathbf{v} = \beta^{-2}(\boldsymbol{\omega} \times \mathbf{r}) \tag{11}$$

so that

$$\beta^2\mathbf{R} = \mathbf{r}$$

$$\mathbf{R} = B^2\mathbf{r} = (\boldsymbol{\sigma} \times \mathbf{V})(Mc^2)^{-1} \tag{12}$$

The "particle" with rest mass μ and velocity $\mathbf{V} > c$ moves on a circle with radius $\mathbf{R}$, while the particle with rest mass m and velocity $\mathbf{v}$ moves in phase with it on a circle with radius $\mathbf{r}$. They are not separate particles, but the single spinning particle can be regarded as composed of them.

It also follows that

$$\mu = im\beta, \qquad M^2 = m^2 + \mu^2 \tag{13}$$

and that the energy of the particle moving on R is

$$\mu\Gamma c^2 = -m\gamma\beta^2c^2 \qquad (\Gamma = i\beta\gamma) \tag{14}$$

The center of mass of $m\gamma c^2$ on $\mathbf{r}$ and $-m\gamma\beta^2c^2$ on $\mathbf{R} = \mathbf{r}\beta^{-2}$ therefore remains fixed at the origin. It is as if an energy $M\gamma^2c^2$ is created virtually on $\mathbf{r}$ and an energy $M\gamma^2\beta^2c^2$ is annihilated on $\mathbf{R}$, leaving the energy Mc^2. The energies combine linearly; the masses, according to (13), quadratically. It is not surprising that a negative energy appears in the structure. We have seen that the center of mass of a point particle with finite spin may lie at some point outside the particle, a situation that is impossible unless some negative energy regions are generated inside the particle as it is compressed to a dimension smaller than σ/Mc.

3. COVARIANT RELATIONS

Equations (1)–(3) may be expressed in relativistically covariant form (see footnote 3):

$$P_\alpha = mcu_\alpha + \dot{\sigma}_{\alpha\beta}u^\beta = mcu_\alpha - \sigma_{\alpha\beta}\dot{u}^\beta$$

$$u^\alpha P_\alpha = mc, \qquad \sigma_{\alpha\beta}u^\beta = 0$$

with

$$u^\alpha u_\alpha = 1, \qquad \cdot = d/ds, \qquad u^\alpha = \dot{x}^\alpha \ (\alpha = 0, 1, 2, 3)$$

Thus, $\dot{u}^\alpha P_\alpha = 0$ and for motion in a general electromagnetic field

$$\dot{P}_\alpha = ef_{\alpha\beta}u^\beta$$

so that $u^\alpha \dot{P}_\alpha = 0$. Thus $u^\alpha P_\alpha = mc$ is a constant throughout any motion caused by electromagnetic forces.

The rest mass M is given by

$$P_\alpha P^\alpha = M^2c^2 = m^2c^2 + \sigma_{\alpha\beta}\dot{u}^\beta\sigma^{\alpha\gamma}\dot{u}_\gamma$$

For $\mathbf{P} = 0$, this reduces to equation (13) with $M = m\gamma^{-1}$.

4. QUANTUM CONDITIONS

The orbital angular momentum is usually defined as $\mathbf{r} \times \mathbf{P}$, which is zero, but here one particle is on orbit $\mathbf{r}$ with momentum $-\mathbf{P}_\sigma$ and the other, with equal and opposite momentum $\mathbf{P}_\sigma$, is on orbit $\mathbf{R}$. The total angular momentum is therefore

$$\mathbf{J} = -\mathbf{r} \times \mathbf{P}_\sigma + \mathbf{R} \times \mathbf{P}_\sigma = (\beta\gamma)^{-2}\mathbf{r} \times \mathbf{P}_\sigma = \boldsymbol{\sigma}$$

from (12). In this representation of the particle, its spin is transformed into orbital motion, the orbital angular momentum of the particle m on the radius $\mathbf{r}$ being

$$\mathbf{L}_m = -\beta^2\gamma^2\boldsymbol{\sigma} \tag{15}$$

while that of the 'particle' μ on the radius $\mathbf{R}$ is

$$L_\sigma = \gamma^2 \boldsymbol{\sigma} \tag{15}$$

so that, as noted, $\mathbf{L}_m + \mathbf{L}_\sigma = \boldsymbol{\sigma}$.

If now we postulate that

$$(L_m)_z = -n\hbar; \qquad (L_\sigma)_z = N\hbar; \qquad \sigma_z = j_z\hbar \tag{16}$$

it follows that

$$n = \beta^2\gamma^2 j_z; \qquad N = \gamma^2 j_z, \qquad N - n = j_z \tag{17}$$

5. THE 3–3 RESONANCE Δ

For $j_z = 3/2$, $n = 1$, and $N = 5/2$ it follows that $\gamma^2 = 5/3$ and $\beta^2 = 2/5$, so that, if $m\gamma c^2$ is identified with the Δ3–3 resonance (which we take to be 1236 MeV), then the emitted particle on R has an energy $E_\sigma = m\gamma\beta^2 =$ 494.4 MeV (cf. $K^+ = 493.65$ MeV). In MeV we then have $mc^2 = 957.40$ (cf. $\eta' = 957.50 \pm 0.24$), $P_\sigma c = m\gamma\beta c^2 = 781.72$ [cf. $\omega(1^-) = 782.0 \pm 0.1$]. The mass Δ is divided into two parts, 494.4 and 741.6 MeV, i.e., in the ratios 2/5 and 3/5. If these are combined quadratically as in equation (13), they define the mass-energy $E' = M'c^2 = (13)^{1/2}/5\Delta = 891.29$ [cf. $K^*(1^-) =$ 892.1 ± 0.3]. Similarly $(E'^2 + E_\sigma^2)^{1/2} = (17)^{1/2}/5\Delta = 1019.23$ [cf. $\phi(1^-) =$ 1019.41]. These results imply that $\eta'^2 = 3\omega^2/2$. The data yield $\eta'^2 =$ $(1.499 \pm 0.001)\omega^2$. Apart from the choice $n = 1$ (which leads to $l = 0$ in the corresponding quantum theory), we have made no assumptions for Δ, although we have used 1236 MeV for its mass instead of the experimental value 1232 ± 2. We have introduced no quarks or gluons or any interactions or parameters. At this semiclassical level we find that the very existence of this spin-3/2 resonance implies the existence of a number of mass-energy levels that lie within 0.15% of the masses of the $K^\pm$, $\eta'(0^-)$, and K^*, ω, and $\phi(1^-)$. However, at this level there is no hint of particle quantum numbers such as strangeness and isospin.

6. MATTER AND ANTIMATTER

Interchange of N and n and change of sign of j_z lead to the same spectrum but with the transformations $m \leftrightarrow \mu$, $\gamma \leftrightarrow \Gamma$, $\beta \leftrightarrow B$, $r \leftrightarrow R$. In the above case, then, $N = 1$, $n = 5/2$ and $j_z = -3/2$ corresponds to $\Gamma^2 = 5/3$ and $B^2 = 2/5$. The orbits are interchanged, suggesting that if m circulates on the inner orbit ($j_z > 0$), the particle is a baryon, and if m lies on the outer orbit ($j_z < 0$), the particle is an antibaryon. In this latter case, m is imaginary and $V < c$, $v > c$. Of course, the sign of j_z is not dependent on the

Table I. Examples of the Transformation m, γ, β, $\mathbf{v}$, $\mathbf{r} \leftrightarrow \mu$, Γ, B, $\mathbf{V}$, $\mathbf{R}$

$$\gamma = -i\Gamma B$$

$$\gamma = (1-\beta^2)^{-1/2} \qquad i\gamma\beta = \Gamma \qquad \Gamma = (1-B^2)^{-1/2}$$

$$v/c = \beta = B^{-1} = c/V$$

$$\mathbf{v} = B^{-2}\mathbf{V}$$

$$\beta^{-2}\mathbf{v} = \mathbf{V}$$

Definitions of $\mathbf{\Omega}$ and $\boldsymbol{\omega}$

$$mc^2 = -\mathbf{\Omega} \cdot \boldsymbol{\sigma}, \qquad \mathbf{\Omega} \cdot \boldsymbol{\sigma} < 0$$

$$-\boldsymbol{\omega} \cdot \boldsymbol{\sigma} = \mu c^2 \Gamma^{-1}$$

Kinetic plus mass energy

$$E = m\gamma c^2 + \mu\Gamma c^2 = (\Gamma^2 B^2 \Omega + \gamma^2\beta^2\omega) \cdot \sigma$$

$$= \mathbf{v} \cdot \mathbf{P} + mc^2\gamma^{-1} = \mathbf{V} \cdot \mathbf{P} + \mu c^2 \Gamma^{-1}$$

$$= \boldsymbol{\omega} \cdot \mathbf{L} - \mathbf{\Omega} \cdot \boldsymbol{\sigma}$$

Kinetic momentum

$$\mathbf{P} = m\gamma\mathbf{v} + \mu\Gamma\mathbf{V} = (\Gamma^2 \mathbf{\Omega} \cdot \boldsymbol{\sigma}\mathbf{V} + \gamma^2 \boldsymbol{\omega} \cdot \boldsymbol{\sigma}\mathbf{v})c^{-2}$$

$$= \gamma^2(\boldsymbol{\omega} - \mathbf{\Omega}) \cdot \boldsymbol{\sigma}\mathbf{v}c^{-2} = \Gamma^2(\mathbf{\Omega} - \boldsymbol{\omega}) \cdot \boldsymbol{\sigma}\mathbf{V}c^{-2}$$

$$M^2 = m^2 + \mu^2$$

arbitrary orientation of an external set of axes. We have taken N and n to be positive so that, from (16) and (17), for $\beta^2 < 1$ and $\gamma^2 > 1$, j_z is positive, opposite to $(L_m)_z$. For $B^2 < 1$ and $\Gamma^2 > 1$, j_z is negative, the same as $(L_m)_z$. In either case j_z has the sign of the z component of the angular momentum of the outer orbit. Transformations from $j_z > 0$ states to $j_z < 0$ states would seem to be impossible because they would require the orbits to interchange, moving both particles through the light barrier. This transformation is described in Table I, with $\mathbf{\Omega} = \boldsymbol{\omega}$ for $\mathbf{P} = 0$.

We note that the baryon is represented by the moving mass m, a parameter that, as shown above, is constant under any electromagnetic force. However, the antibaryon, the same in mass and in the energies of its internal 'partons,' is represented by the fictitious mass μ that was introduced just to make the extra terms in the momentum and energy *look* like momentum and energy. While μ is constant for motion with constant speed, being equal to $im\beta$, it is not at all constant in general. For the antibaryons, $\beta > 1$, but $B < 1$, and μ circulates on the orbit R, which is now the inner orbit. The energy of the mass m on the outer orbit now denotes the energy of an emitted meson. This one asymmetry between baryons and antibaryons could possibly account for the asymmetry between them in the universe. Baryons would appear to be more stable against disintegration by very high energy collisions, since during those collisions μ can change, but m cannot.

For baryons or antibaryons either n or N is half-integral and therefore cannot be represented in the corresponding quantum theory by a single-valued wave function. We may define the integer

$$l'_z \equiv N - n - \frac{1}{2}B = j_z - \frac{1}{2}B$$

so that $l'_z\hbar$ can represent the z component of orbital angular momentum. We then note that interchange of N and n and change of sign of l'_z corresponds to sign changes of both j_z and B.

Another interesting aspect of the transformation from $j_z > 0$ states to $j_z < 0$ states is that, for $\beta < 1$, the inner state (radius r, energy $M\gamma^2c^2$) represents a baryon created and the outer state (radius R, energy $M\Gamma^2c^2$) represents a meson emitted. For $\beta > 1$ and therefore $B < 1$, however, the inner state (radius R, energy $M\Gamma^2c^2$) represents an antibaryon created and the outer state (radius r, energy $M\gamma^2c^2$) now represents a meson emitted. Thus, the symmetry between m and μ, γ and Γ, and β and B not only relates particles and antiparticles to each other, it also links fermions and bosons. This linkage is at a very elementary level, being determined at this level primarily by the accurate theoretical relations between the masses of a fermion and the corresponding boson. The ratio of these boson to fermion masses is $\beta^2 = n/N$ for $j_z > 0$ and therefore $N > n$, and $B^2 = N/n$ for $n > N$.

7. EFFECTIVE POTENTIAL ENERGY

These two "particles" are correlated in their motions, but because they are moving in circles around their common center of mass, they behave *as if* they are attracted to that center with forces $m\gamma\beta^2c^2r^{-1}$ and $\mu\Gamma B^2c^2R^{-1}$. With $r = \beta\gamma\sigma/mc$ and $R = B\Gamma\sigma/\mu c = \gamma\sigma/mc$, it follows that these forces are equal and opposite. Considered as a function of r, the inward force on the particle on that orbit would have to be $mc^2r(r^2 + r_0^2)^{-1/2}r_0^{-1}$ for that particle to stay in orbit ($r_0 = \sigma/mc$), as if that particle were moving in a central field with potential energy

$$V = mc^2(r^2 + r_0^2)^{1/2}r_0^{-1} = m\gamma c^2 \quad (18)$$

This potential energy is constant ($=mc^2$) for $r \ll r_0$, and it increases linearly with r for $r \gg r_0$. It is equal to the relativistically increased mass-energy of the particle for all r. Similarly, the force that would be required to keep the particle μ on the radius R is the same except for sign, because the orbit is β^{-2} as large in radius and the particle energy $-m\gamma\beta^2c^2$ is $-\beta^2$ as much. As a function of R,

$$V = \mu c^2(R^2 + R_0^2)^{1/2}R_0^{-1} = -m\gamma\beta^2c^2 \tag{19}$$

($R_0 = \sigma/\mu c$). However, in differentiating (18) to obtain the force we keep r_0 constant, whereas the force is obtained from (19) by keeping R_0 constant. Here also V is equal to the kinetic-plus-mass energy of the circulating particle, which in this semiclassical picture is negative. The total effective potential energy is Mc^2. The behavior of (18) for large r is that of "infrared slavery," and for small r that of "asymptotic freedom."

8. PAIR CREATION AND ANNIHILATION

For $j = 1/2$, it follows from (17) that $\gamma^2 = 2n + 1 = 2N$, so that if M is equal to the mass of some spin-1/2 baryon (e.g., p), then $m\gamma = M\gamma^2 = (2n+1)M$ and $-m\gamma\beta^2 = -2nM$ as if n pairs $p\bar{p}$ are created on the radius r and annihilated on the radius R. In the original single-particle picture, a charge e circulates on the radius r. With $r = \beta^2 R$ this is equivalent to a charge $\beta^2 e$ on R and a charge $\gamma^{-2}e$ at the origin, the center of charge continuing to circulate on r. In the above case, that would place a charge $e/(2n+1)$ at the origin and the rest on the radius R. However, for a particle of charge e and n pairs, the average charge is also $e/(2n+1)$ [i.e., $e(n+1)/(2n+1)$ from the charges e and $-en/(2n+1)$ from the charges $-e$] and choosing $2n$ particles out of the $(2n+1)$ gives an average charge $2ne/(2n+1) = \beta^2 e$ on R. For $n = 1$, these fractions are $2e/3$, $-e/3$, and $2e/3$.

9. HADRON MASS RATIOS

Table II gives some examples of relations between hadron masses. Here, Mc^2 is the difference between the mass-energy of a baryon or meson and that of a meson that it may emit, their mass ratio being equal to $\beta^2 = n/N$. This energy difference is excited to the virtual state defined by the energy of the particle on the inner orbit, but returns to its normal value with the emission of the particle on the outer orbit. For the special case in which M is the mass of a spin-1/2 particle, the 'particle' on the outer orbit is a pair that has been created on the inner orbit. As the differences between calculations and mass data (mostly within probable error, never

Table II. Examples of Hadron Mass Ratios Compared with Calculated Values of β^2

	n	$\beta^2 = n/n + j_z$	Baryon	Meson	Experimental mass ratio[a]
$j_z = 3/2$	1	$2/5 = 0.400$	Δ	$K^{\pm}(0^-)$	$0.4007 \pm 8 \times 10^{-4}$
	3/2	$3/5 = 0.500$	Ξ^-	$\rho(1^-)$	$0.5005 \pm 3 \times 10^{-4}$
	2	$4/7 = 0.571$	Ω	$\eta'(0^-)$	$0.5725 \pm 3 \times 10^{-4}$
	5/2	$5/8 = 0.625$	Δ	$\rho(1^-)$	0.624 ± 10^{-3}
	3	$6/9 = 0.667$	Ξ^0	$\phi(1^-)$	$0.6655 \pm 1.5 \times 10^{-4}$
	7/2	$7/10 = 0.700$	Ω	$h_1(1^+)$	$0.70 \pm 2 \times 10^{-2}$
$j_z = 1/2$	3/2	$3/4 = 0.750$	Σ^+	$K^{*\pm}(1^-)$	$0.7498 \pm 2 \times 10^{-4}$
	2	$4/5 = 0.800$	Λ	$K^{*\pm}(1^-)$	$0.7994 \pm 2 \times 10^{-4}$
	5/2	$5/6 = 0.8333$	p	$\omega(1^-)$	$0.8334 \pm 2 \times 10^{-4}$
	3	$6/7 = 0.85714$	Σ^+	$\phi(1^-)$	$0.85710 \pm 5 \times 10^{-5}$
	7/2	$7/8 = 0.8750$	Λ	$f_0(0^+)$	$0.874 \pm 3 \times 10^{-3}$
η' series	2	$4/5 = 0.8000$	$\Sigma^-(1/2)$	$\eta'(0^-)$	$0.7996 \pm 3 \times 10^{-4}$
	5/2	$5/8 = 0.6250$	$\Xi^0(3/2)$	$\eta'(0^-)$	$0.6251 \pm 3 \times 10^{-4}$
	3	$6/7 = 0.8571$	$\Lambda(1/2)$	$\eta'(0^-)$	$0.8583 \pm 3 \times 10^{-4}$
			Meson	Meson	
$j_z = 1$	1	$2/4 = 0.500$	$f'_1(1^+)$	$\rho(1^-)$	0.50 ± 10^{-2}
	3/2	$3/5 = 0.600$	$f_1(1^+)$	$\rho(1^-)$	0.60 ± 10^{-2}
	2	$4/6 = 0.667$	$h_1(1^+)$	$\omega(1^-)$	0.67 ± 10^{-2}
	5/2	$5/7 = 0.7143$	$\rho(1^-)$	$\eta(0^-)$	0.714 ± 10^{-3}

[a]Particle Data Group (1990).

greater than 0.25%) are much less than the splitting of multiplets, in Table II we list individual members of each multiplet.

10. SUMMARY

The relativistic classical mechanics that is the basis for this work has been used to explain factors of 2 in magnetic moments, spin–orbit coupling, and Thomas precession, and (with an extra term), to interpret measurements of the anomalous magnetic moments of muons and other particles (see footnote 3). It is noted here that, according to this level of mechanics, the momentum, energy, and angular momentum of a point particle with spin are each equal to the sum of those of two spinless particles moving on two concentric circles with equal and opposite moments (or on a double helix if the momentum is not zero). We have then developed for the relativistic classical mechanics of a particle with spin what de Broglie, Bohr, and Rutherford did so many years ago for the nonrelativistic mechanics of a particle without spin. For example, from equations (4) and (17),

$$Mc^2 \equiv mc^2\gamma^{-1} = j_z\hbar\omega, \qquad N = n + j_z$$

$$E_m \equiv m\gamma c^2 = N\hbar\omega, \qquad E_\sigma \equiv -m\gamma\beta^2c^2 = -n\hbar\omega$$

$$cP_m \equiv m\gamma\beta c = c|P_\sigma| = (nN)^{1/2}\hbar\omega = \hbar c/\lambda\!\!\!^{-}$$

so that, with $r = \beta c/\omega$ and $R = Bc/\omega = c/\beta\omega$

$$r = n\lambda\!\!\!^{-}; \qquad R = N\lambda\!\!\!^{-}$$

The de Broglie relations therefore apply to the individual orbits with their common de Broglie wavelength.

Of course, with very good reason, Bohr and Rutherford introduced the Coulomb potential. Here we have not postulated any potential at all. However, the two equivalent particles move in phase on two concentric circles as if they were moving in a potential the shape of which is derived from the analysis. It has the characteristics of both "asymptotic freedom" and "infrared slavery." The spin is now represented as orbital angular momentum, and the spinning particle automatically contains a number of internal "partons" some of which have energies that are precisely equal to the rest-energies of other particles. This many-particle picture, derived at the semiclassical level from the single-particle picture, therefore offers a basis for describing virtual processes such as pair creation and annihilation and the excitation of baryon states with the emission of mesons, the masses of which appear with remarkable accuracy as simple consequences of the analysis. The complexity of a spinning point particle is thus an immediate consequence of relativistic classical mechanics, and the discreteness of that complexity is a consequence of primitive quantization. There are no phenomenological potentials or postulated particles. The only freedom in Table II lies in the choice of the integer or half-integer n.

APPENDIX. WAVE MECHANICS AND CORRESPONDENCE PRINCIPLE

The Klein–Gordon equation that describes the orbit of particle m on the radius r is

$$[(W - V)^2 - m^2c^4]\psi = -\hbar^2c^2\nabla^2\psi$$

where V is the effective potential energy (18) and W is the total energy, in this case equal to $2V$. The θ- and ϕ-dependent components of the wave function are described by spherical harmonics in the usual way, and

$$\frac{d^2R}{dr^2} = \left(\lambda_c^{-2} - \frac{(W-V)^2}{\hbar^2c^2} + \frac{l(l+1)}{r^2}\right)R$$

where $R = r\psi(r)$ and $\lambda_c = \hbar/mc$. The factor of R on the right-hand side has a maximum for

$$r^3 = \frac{l(l+1)\hbar^2c^2}{(W-V)V'} = \frac{2l(l+1)\lambda_c^2}{d\gamma^2/dr} \tag{A1}$$

where γ is defined by the classical relation

$$r = \gamma\beta j_z\lambda_c$$

or

$$\gamma^2 = \frac{r^2}{j_z^2\lambda_c^2} + 1$$

Thus the maximum of $R^{-1}\, d^2R/dr^2$, corresponding to a minimum in the effective potential for radial motion, occurs for

$$r^2 = r_{\text{min}}^2 = [l(l+1)]^{1/2} j_z\lambda_c^2 \tag{A2}$$

This corresponds to the classical quantized radius with $\gamma^2\beta^2 = nj_z^{-1}$, and $n \to l$ for large l.

REFERENCES

Aldinger, R. R., *et al.* (1983). *Physical Review D*, **28**, 3020–3031.
Bhabha, H. J., and Corben, H. C. (1941). *Proceedings of the Royal Society of London A*, **178**, 273–314.
Corben, H. C. (1961*a*). *Physical Review*, **121**, 1833–1839.
Corben, H. C. (1961*b*). *Nuovo Cimento X*, **20**, 529–541.
Corben, H. C. (1984). *Physical Review D*, **30**, 2683–2689.
Corben, H. C. (1993). *American Journal of Physics*, **61**, 551–553.
Hughes, V. W., and Wu, C. S., eds. (1977). *Muon Physics*, Vol. 1, *Electromagnetic Interactions*, Academic Press, New York, p. 59.
Moller, C. (1952). *The Theory of Relativity*, Oxford University Press, Oxford, pp. 170–173.
Particle Data Group (1990). *Physics Letters B*, **239**.